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Two Degrees of Freedom in the Control of a DC-DC Boost Converter, Fuzzy Identified Explicit Model in Feed-forward Line

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Abstract We present a novel approach to the fuzzy control of a DC-DC Boost Converter. Using heuristic partitioning of the main control parameters and focusing on global knowledge of the open-loop, stable system's equilibriums, the new method is developed based on an offline fuzzy identification of the steady-state duty cycle. The explicit and the fuzzy identified global model of the duty cycle robustly contribute to the system's stability, even in the presence of large changes to the process parameters. In comparison with the analytically derived duty cycle using two different methods, the identified model prediction of an infinity horizon duty cycle shows better precision. These results are achieved in an analysis of the converter's hybrid-simulation model where the assumptions made in the mathematical modelling are minor in comparison with similar assumptions in physical examples. The steady-state error compensation relies on the optimized PI controller, which is independently constructed and involved in the final Two-Degrees-of-Freedom (TDOF) controller. The successful simulation results agree with the robustness and present a DC-DC converter with stable operation, even in the dynamic exchange of the DCM (Discontinuous Conduction Mode) and CCM (Continuous Conduction Mode). The method is widely applicable as it minimizes the real time of processing and avoids over-determined solutions.

Keywords DC-DC boost converter · Hybrid modelling · Fuzzy identification · Robust control of nonlinear dynamical system · Explicit Fuzzy Model Predictive Control (EFMPC) · Two Degrees of Freedom (TDOF)

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1 Introduction

Even though the control of DC-DC converters has been very well examined from different aspects with respect to control techniques [1], model predictive control (MPC) remains as one of the most systematic and frequently used methods [2]. The wide range of applications for all type of pulsed-energy converters (PECs) dictate the main features of control algorithms and as a result place constraints on the overall solutions. MPC systematically handles the problem of constraints, but at the same time puts an extra burden on the processor's time of execution and certainly explains the method's main drawback, which relates to the complexity of computation. The complex algorithms then necessarily affirm a new nonlinear phenomena scenario in the transition time of the control and the system's steady state, so harming the stability of the system. Most of the problems addressed in previous work on the control of DC-DC converters are consisted in terms of the uncertainties in the mathematical modelling of PECs.

Hybrid modelling and control represent the state of the art in terms of the exploration and design of PECs [1]. The authors in this reference give a short overview and a comparison of the profiled hybrid approaches, mostly based on *LMI,* and optimization by convex programming or a *Lyapunov-function* motivated stability optimization. Systematic control approaches implement the *LQ* optimization [3] or H_∞ control [4] in a sense to provide a certain robustness to the plant uncertainty and all are based on a linearized sampled data model. On the other hand, relaxing the dynamic programming introduces a complexity to the optimization and opens up the well-know Lyapunov-function selection problem [5], with a solution in time-limiting convergence. Of no less importance is to consider the overall applicable approaches based on current control and known as *sliding mode control* [6, 7]. In contrast to the experience in these studies we are again emphasizing the well-known problem of modelling systems with discontinuities, but now also bearing in mind the applicability of standard control methods.

In one switching period T_S we receive multiple changes to the semiconductors' positions and, necessarily, the circuit's topology. Most of the previous modelling [8–10] are based on the successive adding of the piecewise affine models or forming an average-switched model well presented in the publication of Middlebrook and Ćuk [11]. The modelling solution is based on the perturbation method, and it is valid for small signal values. Furthermore, the same authors, together with Erickson [12], provide more precise modelling, called *large signal* modelling, which is also applicable for robust applications. In today's control solutions more interest is put on nonlinear phenomena exclusion [8–10, 13, 14], which necessarily leads the mathematical discussion to well-posedness and solution existence in the modelling of hybrid systems [15]. From modern mathematical aspects, the modelling of DC-DC converters leads to complementarity formalism [15, 16] and has to be treated accordingly. It gives a better insight into the switching problem and a qualitative and quantitative system-state trajectories' pattern [16]. A complementarity framework is used in sliding-mode control solutions [17], but with no wider control applicability [16]. The modelling problem certainly becomes more complicated by assuming real circuits, where the ideal switches are excluded and an unpredicted estimated serial resistance (ESR) is encountered, combined with a different system's parameter changes.

To use the standard and developed control methods, but also to anticipate the possibility of nonlinear phenomena scenarios, we suggest a new control based on system identification and the construction of a global dynamic or steady-state system model. In this work we will concentrate on the global steady-state model solution. Our method is built on the basis of three-fold approach. First, transfer the complexity of the computation to the offline regime. Second, concentrate the major part of the examination on the equilibriums in a global and robust sense. Third, by assuming a fully measurable system, involve the state variables as transformed average values. The last of these arises from the main objective in most DC-DC converters' control algorithms, which is the control of the output voltage's average level and not the output voltage's signal trajectory. Following this, the mathematical framework will not be exact and the previously mentioned problem of *differential inclusion* and *complementarity formalism*, but rather solutions in the pseudo norm vector space. Theoretically, it is strongly supported in [18], and elementarily connected to the approximation and smoothing operation of disjoint sets in the Lebesgue space.

This approach will emphasize MPC more as a methodology than as a strict control technique and agree with the statements in [19]. It is focused on output voltage control and has a correlation with most of the mentioned hybrid modelling approaches, but mostly as an agreement with objectives in the current PEC control development. MPC [20] is opening the discussion on duty-cycle modelling and approximation with ν-resolution. The standard analytical modelling does not give a uniformly spread error for the duty-cycle approximation in the constrained range *[0,1]*. With respect to this, our work avoids a strict resolution that is relative and strongly depends of the *Fuzzy Model Membership* construction. With this approach, a graphical model is not a polyhedron of the piece-wise affine systems with sharp edges, but rather by avoiding edges and softening uncertain transits, it is a complex foliation. Similarly, a correlation with dynamical programming [5] and the relaxation method produces the same findings in relation to the stability of the optimisation algorithms, the prediction of the convergence time and the stopping problem. The last of these fortifies our opinion of the necessity for offline optimization, even when knowing a current processor's capabilities. The idea is conceived in previous studies by different authors, e.g. [2], but also with a different mathematical framework [21].

The Offline Fuzzy Identification presented here is a global duty-cycle reconstruction, or MISO model as an atlas of the steady-state mappings or graphically a folium related to different process parameters. Any selection of the measured input variables on the input universes of discourse is associated throughout the fuzzy engine with a single and unique steady-state duty cycle. The identified model is the *Global and Explicit Model*, which then constructs the bases for a MPC algorithm. This approach differs from the classic preceding horizon MPC as it gives a time-invariable solution that is more similar to the infinity horizon solution, hence being explicitly driven without the necessity for an inverse function calculation. Also different than a classic fuzzy control [13, 14], this paper supports the heuristic approach that implements the fuzzy identification, and after a modelling, moved from the strict analytical framework built on the piecewise linearity. The fragmentation of the MPC method leads us to the construction of the Two Degrees of Freedom Control, where the feed-forward line selects the explicit fuzzy MPC's (EFMPC) based steady duty cycle, further corrected by the small signal PI optimized controller. The complete work is done on the MATLAB simulation platform [22], which does not limit the applicability of the method, but rather proves the method even in ideal situations, where an approximation in the modelling is more precise than with known analytical methods.

This paper is organized as follows. Section 2 starts with a presentation of the hybrid modelling of the boost DC-DC converter and explains a basic problem in the analytical system examination. Section 3 explains the fuzzy model identification. Section 4 presents the applied control method, followed by the simulation results. Finally, Section 5 is a short conclusion and a description of future developments.

2 Hybrid Simulation Model

An example is taken from literature [9], with all its numerical values, in order to be able to compare the results with previous research.

For a principal part of the electronic circuit on Fig. 1, apart from the pulse-width modulator (PWM) and controller with its set point s, using Kirchhoff's voltage and current laws, we form the ordinary differential equations (ODEs) $\dot{z} = f(z) + g(z)u$. Our state vector is $z = [v_C \;\; i_L]^T$ and the input will be a voltage source E.

The mathematical model is developed on the assumption that semiconductors are ideal switches with no voltage drops and the inductivity has no equivalent series resistance (ESR). The system is sequentially driven over three modes related to the position

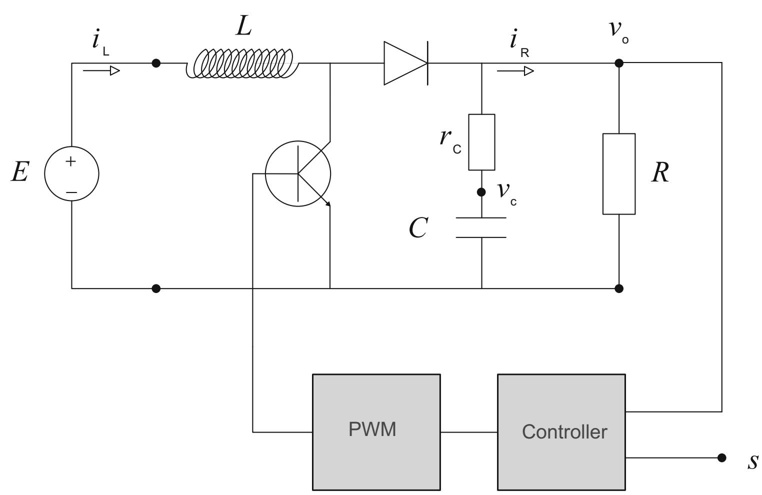


Fig. 1 Typical DC-DC boost converter, voltage controlled

of the switches and this applies to the three sets of state-space equations for a different circuit topology [9]

$$\dot{z} = \boldsymbol{A}_i z + \boldsymbol{B}_i E \qquad i \in [1, 2, 3] \quad . \tag{1}$$

During the period T_S we can stack the consecutive solutions for each time $t_{i,k}$ interval and form the transition map for the complete period, by knowing that $t_{i,k} + t_{2,k} + t_{3,k} = T_S$.

The analytical examination can be continued as a way to further develop the difference equation. This leads to a procedure of neglecting the higher-order powers and approximations of the constructed transition matrix. So, a successive substitution with the involvement of the Taylor power series approximation evolves in a tedious input/output system representation, but still with the involvement of approximations and the ability to examine only simple switching algorithms [9, 10]. Furthermore, even if a simple switching algorithm is selected, the analytical definition of the duty cycle becomes a transcendental mathematical problem and it can only be solved by numerical methods.

Hence, from the side of nonlinear dynamical system examinations, a general expression has to evolve in $\dot{z} = f(z, d) + g(z, d)u$ for the duty cycle $d = \frac{t_{1,k}}{T_S}$ as a control signal of closed-loop control and the scalar input signal in the process.

A previously explained theory physically leads in the hybrid modelling and examination of a system in its complex overall and natural form.

The nonlinear state-space expression of a DC-DC boost converter is given by:

$$\dot{z}(t) = \begin{cases} \boldsymbol{A}_1 z(t) + \boldsymbol{B}_1 E(t) & kT_S \le t \le kT_S + t_{1,k} \\ \boldsymbol{A}_2 z(t) + \boldsymbol{B}_2 E(t) & kT_S + t_{1,k} < t \le kT_S + t_{1,k} + t_{2,k} \\ \boldsymbol{A}_3 z(t) + \boldsymbol{B}_3 E(t) & kT_S + t_{1,k} + t_{2,k} + kT_S < t \\ & \quad < (k+1)T_S \end{cases}$$
$$k = 0, 1 \ldots \infty \tag{2}$$

in DCM or

$$\dot{z}(t) = \begin{cases} \boldsymbol{A}_1 z(t) + \boldsymbol{B}_1 E(t) & kT_S \le t \le kT_S + t_{1,k} \\ \boldsymbol{A}_2 z(t) + \boldsymbol{B}_2 E(t) & kT_S + t_{1,k} < t < (k+1)T_S \end{cases}$$
$$k = 0, 1 \ldots \infty \tag{3}$$

in CCM of DC-DC converter.

The presented models are exact or a complete system physical representation, and also a source for the construction of the simulation model in Fig. 2. At this point further examinations will follow based on the numerical methods in the simulation and control design of the predictive control algorithm, and in contrast to most of the known, developed methods they will not consist of piece-wise linear expressions. By knowing that the top objective of the following work is the control of the output voltage, which is assumed as a DC signal, then the physical approach leads us to the selection of root mean square (RMS) measured values. Now the new state variables cause a mathematical transformation of the original state space (1) to the

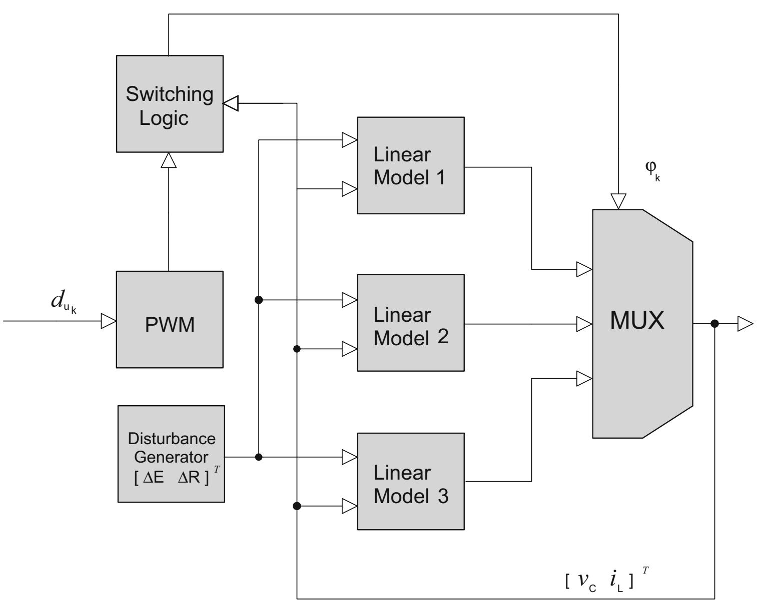


Fig. 2 DC-DC boost converter, hybrid simulation model

pseudo-norm vector space, based on assumptions that a new state vector $\bar{z} = \left[\bar{v}_0, \bar{i}_L\right] \in \mathbb{L}^2$ (Lebesgue), as a product of the numerical integration methods with an approximate solution in the system discontinuity. The RMS measurement of the original state space variables is done in T_S time period.

3 Fuzzy Identification and Modelling

A new pseudo norm $\| \cdot \|$ on vector space $(V, \| \cdot \|)$

$$\| z \|_{\mathbb{L}^p} = \frac{1}{t} \left(\int |z|^p dt \right)^{\frac{1}{p}}, \; z \in \mathbb{L}^p \tag{4}$$

will be derived from the simulation process of the hybrid mathematical model and a numerical integration based on the explicit Runge-Kutta (4, 5) method and developed in the “ode45(Dormand-Prince)” MATLAB [22] for $p = 2$.

The space transformation filters out the high-frequency nonlinearities or the high-scaled oscillations. In the continuation, a pseudo norm space $(V, \| \cdot \|)$ will be further transformed to the pseudo-Banach subspace of an augmented dimension.

The simulation process Q of the hybrid system (2) will expand the origin three-dimensional space including time to the six-dimensional space $Q : \mathbb{R}^3 \to \mathbb{R}^6$. That is possible by knowing that our physical system is state measurable. The extra measured process parameters are E as the input voltage source to the converter, the output current i_R, the control variable to process $d_u = (1-d)0.66 \; for \; d \in [0.1]$ scaled related to the Pulse Width Modulator (PWM). The measurement of the i_R and together with the controlled voltage value will reconstruct the converter’s load.

From the simulations we know that the process is open-loop stable. Even when transformed, the system still preserves its nonlinear dynamical characteristics. With the intention to predict the stable control parameter d_u, our work will concentrate on an examination of the stable steady state of a DC-DC boost converter.

A mapping Q derived the trajectories over the six-dimensional pseudo-Banach space and opens up ability for an orthogonal slicing of the new vector space $(V^6, \| \cdot \|)$ to the tangent vector space $(V_1^6, \| \cdot \|) \subset (V^6, \| \cdot \|)$. Geometrically, $(V^6, \| \cdot \|)$ is a smooth manifold $\boldsymbol{M}_1$, which consists of the targeted six-dimensional tangent space $\boldsymbol{T}_{\hat{x}_0}\boldsymbol{M}_1$. The steady and stable state of the converter is assigned as $\hat{\boldsymbol{x}}_0 \in \boldsymbol{M}_1$. The vector of transformation $\boldsymbol{\tau} = \left\{ \frac{\partial}{\partial t}\Big|_{\hat{x}_0}, \frac{\partial}{\partial \bar{i}_L}\Big|_{\hat{x}_0}, \frac{\partial}{\partial \bar{V}_C}\Big|_{\hat{x}_0}, \frac{\partial}{\partial E}\Big|_{\hat{x}_0}, \frac{\partial}{\partial R}\Big|_{\hat{x}_0}, \frac{\partial}{\partial d_u}\Big|_{\hat{x}_0} \right\}$ is a natural base of $\boldsymbol{T}_{\hat{x}_0}\boldsymbol{M}_1$, and $\boldsymbol{T}_{\hat{x}_0}\boldsymbol{M}_1 = \boldsymbol{\tau} \mathrm{o} \boldsymbol{M}_1\big|_{\hat{x}_0}$. Throughout the tangent space $\boldsymbol{T}_{\hat{x}_0}\boldsymbol{M}_1$ we pull an affine surface orthogonally on the first coordinate of $\boldsymbol{T}_{\hat{x}_0}\boldsymbol{M}_1$ kernel. The so gained surface $\boldsymbol{S}$ consists of the system steady states. The trajectories driven from the simulation process Q intersect the surface $\boldsymbol{S}$ at particular points $\hat{\boldsymbol{x}}_{0,i}$ for $i = 1, \ldots.n$ and n is the number of the final and the time filtered test samples. Because n is a limited number $n < \infty$, our surface $\boldsymbol{S}$ is not dense and it applies for an interpolation. Our transformed original space is now based on affine functions, and by the employment of the identification method; we construct the modelled surface with its minimal error to the representatives $\hat{\boldsymbol{x}}_{0,i}$ of the physical surface.

The main task of the following work is the mathematical definition of a mapping $\boldsymbol{\psi}\left(V_1^6\right) : \mathbb{R}^{5-1} \to \mathbb{R}^{2-1}$ and the construction of the explicit fuzzy model of the control signal d_u, which guarantees a true and predicted system steady state as a consequence of the vector of a measured and $\| \cdot \|_2$ values $\bar{\boldsymbol{x}} = \left[\bar{v}_C \; \bar{i}_L \; \bar{i}_R \; E\right]^T$.

The objectively accepted results of the mapping identification could be reached only by thoughtful selection of the measured process data. This task, by examination of the quantitative system dynamical behaviour, has to exclude always-possible preliminary conclusions that mostly lead to severe model/process errors. In order to support that approach, in this paper we involve the process excitation only with a random pattern. The vector of the process changes $\boldsymbol{\xi}(kT_\infty) = [E(kT_\infty) \;\; R(kT_\infty)]$ originated by the MATLAB white noise and random function together with the excitation duty cycle form the overall input vector $\boldsymbol{u}(kT_\infty) = [d_u(kT_\infty) \mid \boldsymbol{\xi}(kT_\infty)]$. Figure 3 expresses the simulation principle where the discrete time kT_∞ is selected to preserve the steady-state measurement, afterwards

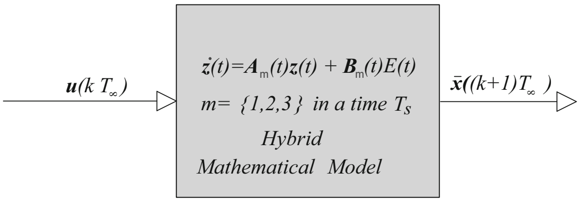


Fig. 3 Mathematical model simulation principle of construction the system-scanned database

resulting in the database and forming the identification training data set.

3.1 Fuzzy Model Identification

Based on the heuristic assumption that there exists a deterministic and unique mapping in the pseudo Banach space $(V^6, \|.\|)$ we form the fuzzy identified model $\mathbf{F}(V_1^6) : \mathbb{R}^{5-1} \rightarrow \mathbb{R}^{2-1}$ where in the process the steady state holds the expression for the duty cycle

$$d_u = \mathbf{F}(\hat{\mathbf{x}}) \tag{5}$$

The input vector to the fuzzy mapping will be $\hat{\boldsymbol{x}} = \left[\bar{v}_o \; \bar{i}_L \; E \;\; R\right]^T$, partly simplified from an $\bar{\boldsymbol{x}}$ with the assumption that $v_c \approx v_o$ and the implementation of $R = \frac{\bar{v}_o}{\bar{i}_R}$, $\bar{i}_R > 0$. Generally, mapping is a function of the input vector $\hat{\boldsymbol{x}} = \boldsymbol{x}$ (in the following assigned as $\boldsymbol{x}$ for reasons of simplicity) defined by its parameters and hence

$$d_u = y = f(\mathbf{x}|\boldsymbol{\theta}). \tag{6}$$

In equation (6) the $\boldsymbol{\theta} = \{\boldsymbol{a}, \boldsymbol{c}\}$ denotes the set of fuzzy model parameters , and in our example

$$\boldsymbol{a} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{b,1} & a_{b,2} & a_{b,3} & a_{b,4} & a_{b,5} \end{bmatrix}. \qquad \boldsymbol{c} = \left[\, \boldsymbol{c}_1 \;\; \boldsymbol{c}_2 \;\; \cdots \;\; \boldsymbol{c}_b \,\right] \tag{7}$$

The constant b is a number of rules in the fuzzy rule base.

For all systems [23] if there is a function

$$g : \quad \tilde{\boldsymbol{X}} \rightarrow \tilde{\boldsymbol{Y}}$$

that $\tilde{X} \subset \mathbb{R}^l, \tilde{Y} \subset \mathbb{R}$ then with the process of identification we approximate the mapping g in the way that

$$g(\boldsymbol{x}) = f(\boldsymbol{x}\,|\boldsymbol{\theta}) + e(\boldsymbol{x}) \quad . \tag{8}$$

The approximation $f(\boldsymbol{x}|\boldsymbol{\theta})$ of a physical system was derived from examinations of the training data set

$$\boldsymbol{G} = \left\{\left(\boldsymbol{x}^1, d_u^1\right), ..., \left(\boldsymbol{x}^M, d_u^M\right)\right\} \subset \tilde{\boldsymbol{X}} \times \tilde{\boldsymbol{Y}}$$

constructed by M data pairs of the steady-state representatives, from a complete data set gained in the simulation Q and corresponding to (5).

In this study the selected C-means clustering method will iteratively minimize the distance

$$J = \sum_{i=1}^{M}\sum_{j=1}^{b}(\mu_{ij})^p \left\|\boldsymbol{x}_i - \boldsymbol{c}_j\right\|^2 \tag{9}$$

from a bonding data representative center vector $\boldsymbol{c}_1, \boldsymbol{c}_2..., \boldsymbol{c}_b$ in vector space $(V_1^6, \|.\|) \subset (V^6, \|.\|)$, of our predefined universes of discourses. The process of clustering will be performed on M data pairs of $\boldsymbol{G}$.

The parameter p is the so-called "fuzziness factor" [23], which determines the factor of overlap in-between clusters and μ the grade of membership.

Accordingly in this study the selected fuzzy model is the Takagi-Sugeno MISO model, which consists of the rule base, presented with an equation

if $\boldsymbol{H}^j$ **then** $g_j(\boldsymbol{x})$

where $\boldsymbol{H}^j$ denotes the fuzzy set

$$\boldsymbol{H}^j = \left\{(\boldsymbol{x}, \mu_{\boldsymbol{H}^j}(\boldsymbol{x})) : \boldsymbol{x} \in \tilde{\boldsymbol{X}}_1 \times ... \times \tilde{\boldsymbol{X}}_n\right\}$$

and

$$g_j(\boldsymbol{x}) = a_{j,0} + a_{j,1}\,x_1 + ... + a_{j,4}\,x_4$$

for $j = 1, 2, ...b$.

The complete fuzzy function is given by

$$f(\boldsymbol{x}\,|\boldsymbol{\theta}) = \frac{\sum_{j=1}^{b}\left(a_{j,0}+a_{j,1}x_1 + ... + a_{j,4}\,x_4\right)\mu_{\boldsymbol{H}^j}(\boldsymbol{x})}{\sum_{j=1}^{b}\mu_{\boldsymbol{H}^j}(\boldsymbol{x})}. \tag{10}$$

As the clustering method does not tune the complete fuzzy parameters $\boldsymbol{\theta}$ but only $\boldsymbol{c}$, a consequence function parameters $\boldsymbol{a}$ will be defined by the least-squares method

$$\begin{aligned} &\boldsymbol{a}_j = \left(\boldsymbol{X}^T\boldsymbol{W}_j^2\boldsymbol{X}\right)^{-1}\boldsymbol{X}^T\boldsymbol{W}_j^2\boldsymbol{Y} \\ &\boldsymbol{X} = \begin{bmatrix} 1 & \cdots & 1 \\ \boldsymbol{x}_1 & \cdots & \boldsymbol{x}_M \end{bmatrix}^T \qquad \boldsymbol{Y} = \left[d_{u,1}, \cdots, d_{u,M}\right]^T \\ &\boldsymbol{W}_j^2 = \left(diag\left(\left[\mu_{1j}, \cdots, \mu_{Mj}\right]\right)\right)^2 \end{aligned} \tag{11}$$

and again in order to minimize the cost function

$$\begin{aligned} &J_j = \sum_{i=1}^{M}(\mu_{ij})^2\left(d_{u,i} - \left[1, \boldsymbol{x}_i^T\right]\boldsymbol{a}_j\right)^2 \\ &j = 1, 2, ..., b. \end{aligned} \tag{12}$$

Accordingly, all the identification processes can be briefly presented in the following algorithm steps:

1. *Simulation of the physical system (Hybrid simulation model) excited with* $\boldsymbol{u} = [d_u \quad \boldsymbol{\xi}]$
2. *Forming of M data pairs of the training data set* $\boldsymbol{G}$
3. *Definition of* μ_{ij} *the new grades of membership* $(p = 2)$ *by the C-means clustering*

$$\mu_{ij} = \left[\sum_{m=1}^{b} \frac{|\boldsymbol{x}_i - \boldsymbol{c}_j|^2}{|\boldsymbol{x}_i - \boldsymbol{c}_m|^2}\right]^{-1} \quad i = 1, ..., M$$
$$j = 1, ..., b \quad \boldsymbol{c} = [\boldsymbol{c}_1,\ \boldsymbol{c}_2, ..., \boldsymbol{c}_b] \quad initially\ selected \tag{13}$$

4. *Definition of* $\boldsymbol{c}_j$ *the new centres by the C-means clustering*

$$\boldsymbol{c}_j = \frac{\sum_{i=1}^{M} \boldsymbol{x}_i \mu_{ij}^2}{\sum_{i=1}^{M} \mu_{ij}^2} \tag{14}$$
$$j = 1, ..., b$$

5. *Definition of* $\boldsymbol{a}_j$ *the consequence parameters by the weighted least-squares method*
6. *Defuzzification (10) by implementation of the grade function*

$$\mu_{\boldsymbol{H}^j}(\boldsymbol{x}) = \left[\sum_{m=1}^{b} \frac{|\boldsymbol{x} - \boldsymbol{c}_j|^2}{|\boldsymbol{x} - \boldsymbol{c}_m|^2}\right]^{-1}$$
$$j = 1, ..., b \quad \boldsymbol{c} = [\boldsymbol{c}_1,\ \boldsymbol{c}_2, ..., \boldsymbol{c}_b] \quad c - means\ tuned$$

The constructed fuzzy model here is the mapping, which transfers a converter's parameters from the input universes of discourse

$$\begin{aligned} \bar{v}_o &= x_1 \in \tilde{\boldsymbol{X}}_1 = [0\,V, 700V] \\ \bar{i}_L &= x_2 \in \tilde{\boldsymbol{X}}_2 = [0A, 1030A] \\ E &= x_3 \in \tilde{\boldsymbol{X}}_3 \ = [10V, 16V] \\ R &= \frac{\bar{v}_o}{\bar{i}_R} = x_4 \in \tilde{\boldsymbol{X}}_4 \ = [10\Omega, 32\Omega] \end{aligned}$$

to the output universe of discourse or simply a duty cycle

$$d_u = y \quad \in \tilde{\boldsymbol{Y}} \ = [1, 5\ \%\ ,\ 98, 5\ \%] = [0.65\ ,\ 0.01]\,.$$

The fuzzy rule base consists of $b = 33$ rules, and the fuzzy parameters $\boldsymbol{\theta} = \{\boldsymbol{a}, \boldsymbol{c}\}$ were reconstructed and based on the knowledge gained by $M = 635$ pairs of $\boldsymbol{G}$.

It can be now presented geometrically as an invariant foliation in the tangent subspace $\boldsymbol{T}_{\hat{x}_0}\boldsymbol{M}_1$. For an understandable graphical presentation, it is drawn in three-dimensional space formed by the original and appreciated dependence of a duty cycle d_u from the coil current i_L and the output voltage v_0, while the remaining dimensions are fixed in the $\hat{\boldsymbol{x}}_0$ points of their constrained universes. Figure 4 expresses the sliding effect in the three-dimensional space of the original nonlinear dynamical system in its equilibriums influenced by a source voltage change. In a similar way, Fig. 5 shows the sliding effect with a load change.

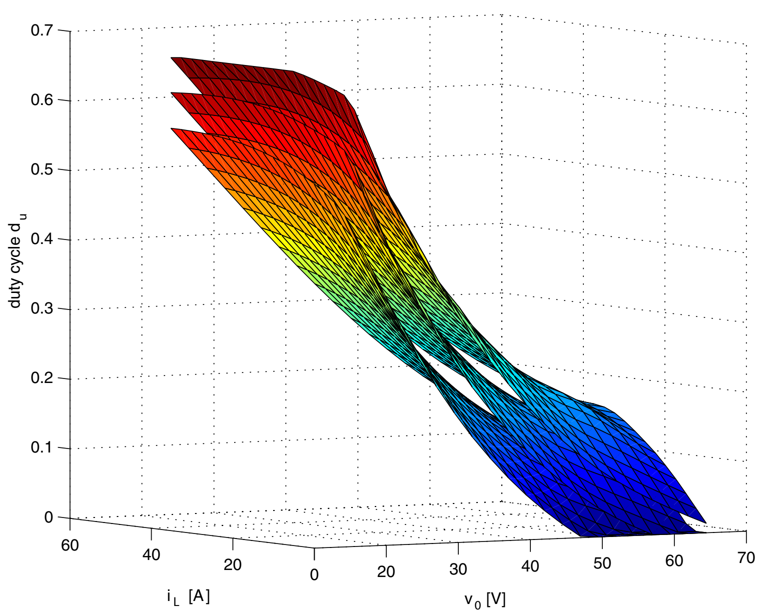
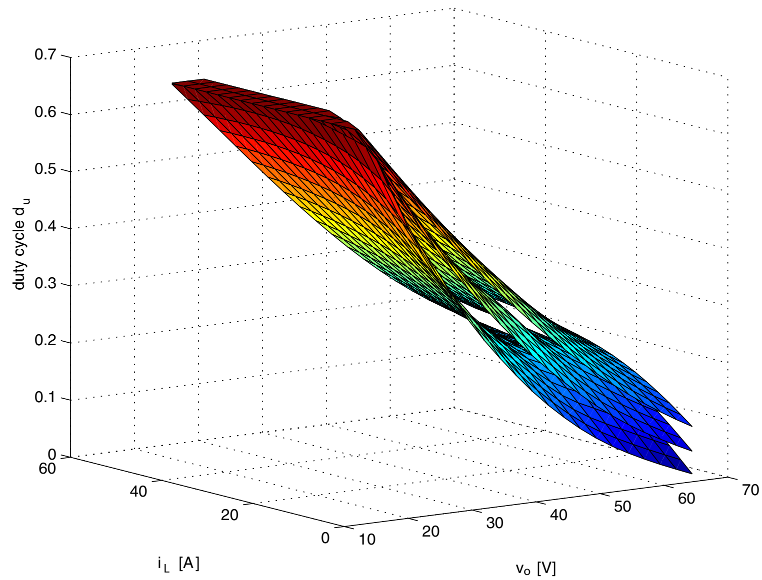


Fig. 4 Explicit Fuzzy Model invariant foliation for step changes of E=10, 13, 16 V (source voltage) and R= 12.5Ω

Fig. 5 Explicit Fuzzy Model invariant foliation for step changes of load R=12.5, 20, 30 Ω and E=16 V



3.2 Fuzzy Model Evaluation

In order to evaluate the fuzzy modelling and achieved results, the evaluation test will be done in such a way that the physical system or the hybrid simulation model in Fig. 2 will be excited with a ramping duty cycle, from 0 % to 100 %, varying as discrete input in the time nT_∞, while the source voltage and the load resistance are fixed. The received data, as the testing data set, will now be the input data to the explicit fuzzy model. The resulting outputs from the explicit model testing (10) will then be back compared with the original excitation duty cycle of the physical model.

Approximation error from the explained test

$$e_n = d_{fuzzy,n} - d_n \tag{15}$$

for n^{th} the data equilibrium set is further evaluated by comparing with the analytical results from [9] and the results based on the average switched method.

Figure 6 is a graphical presentation of the fuzzy model testing results, compared with the real excitation duty cycle. The graph is drawn for one combination of the fixed load and the source voltage, which means that a similar test can be done for a different combination of the fixed process parameters.

For a same data in Fig. 6, a comparison of the results has been made to the analytically calculated duty cycles on two different ways. One calculation is based on small signal values and the averaged model, and the other is based on the stroboscopic Poincaré map analytically derived in [9]. Figure 7 shows all the evaluations together. We see that the stroboscopic Poincaré map approximation

$$v_o(k+1) = \alpha v_o(k) + \frac{\beta d(k)^2 E^2}{v_o(k) - E}$$

$$\alpha = 1 - \frac{T_S}{C(R+r_C)} + \frac{T_S^2}{2C^2(R+r_C)^2}$$

$$\beta = \frac{RT_S^2}{2LC(R+r_C)}$$

$$d_{poin} = \sqrt{\frac{(1-\alpha)(s-E)s}{\beta E^2}} \tag{16}$$

or the averaged system method approximation, developed for the DC-DC boost converter in this investigation

$$d_{lin} = \frac{(E-s)(R+r_c)}{sR} \tag{17}$$

are providing globally less accurate results in the steady-state duty-cycle prediction than the one resulting from the fuzzy model (5),(10).

If we calculate the arithmetically averaged error in all three cases, hence

$$\bar{e}_{Poin} = 0.3592 \qquad or \quad 53.61\ \%$$

$$\bar{e}_{lin} = 0.0149 \qquad or \quad 2.22\ \%$$

$$\bar{e}_{fuzzy} = 0.0079 \qquad or \quad 1.17\ \%.$$

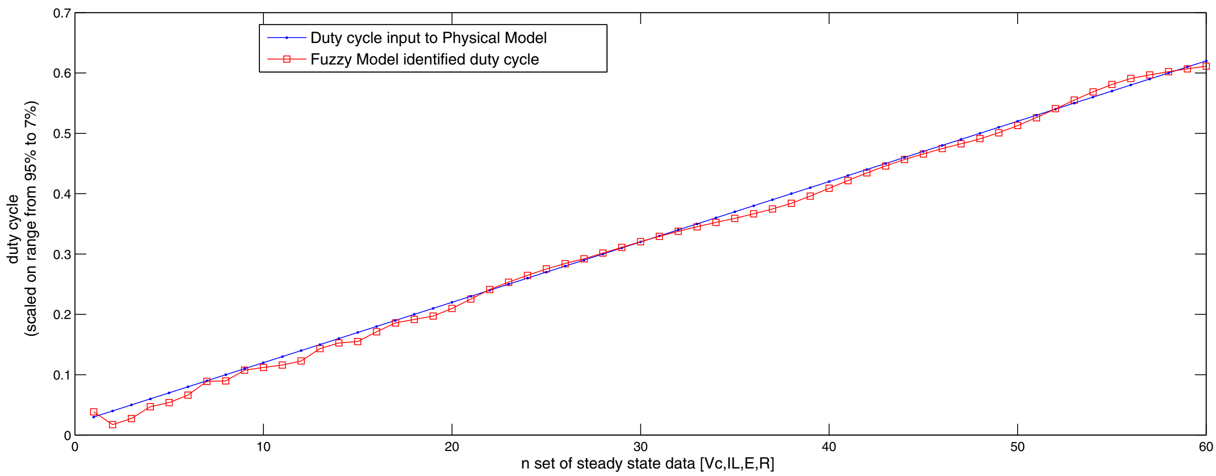


Fig. 6 Comparison of ramping duty cycle to Physical Model and reconstructed duty cycle by Explicit Fuzzy Model, while constant E=16V and R= 12.5Ω

Figure 7 already explains a very inaccurate Poincaré stroboscopic map approximation of the steady-state duty cycle for a complete range of duty cycles. But if we transfer a comparison in the limited range of interest [*16,50*] the VDC of the reference voltage, or only the DCM converter's operation, then the results are more comparable. A stroboscopic map [9] is derived for the converter DCM and it is expected to be non applicable for a complete range of DC-DC converter operations. The methods driven in this work that are the same as the averaged model approximation are the global methods and comparable for a quantitative examination of the physical system. So if we limit the range of interest to that mentioned, the following results were found:

$$\bar{e}_{Poin} = 0.0132 \qquad or \quad 1.97\ \%$$

$$\bar{e}_{lin} = 0.0199 \qquad or \quad 2.97\ \%$$

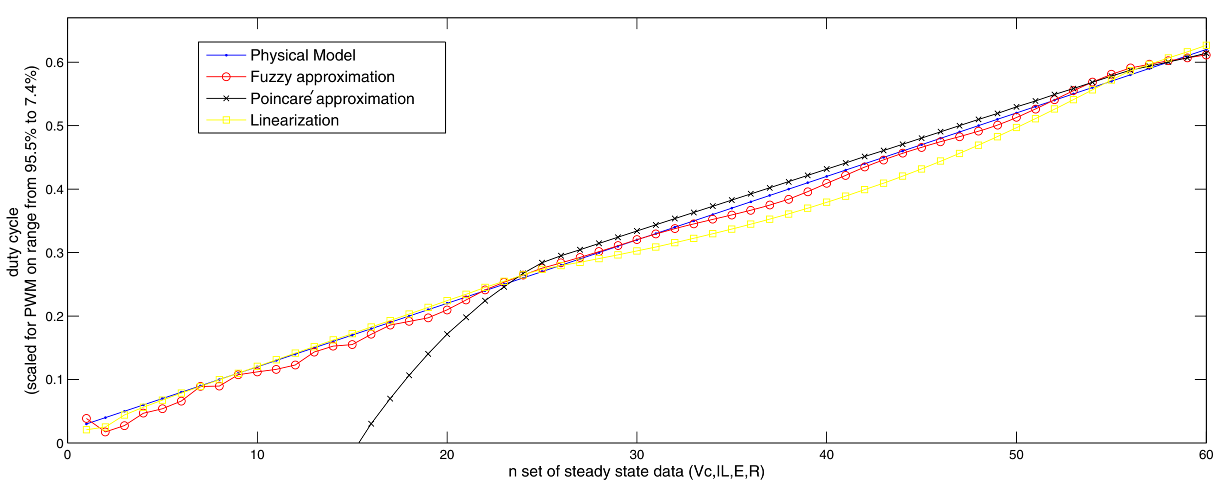


Fig. 7 Comparison of three methods of approximation with a physical model steady-state duty cycle while *constant E=16V and R=12.5 Ω*

$$\bar{e}_{fuzzy} = 0.0065 \qquad or \quad 0.97\,\%.$$

Now, the results of all the methods are comparable and still the fuzzy model is the best approximation of the steady-state duty cycle, and it will be used in a duty-cycle prediction of the control algorithm for the infinity time horizon.

4 Two Degrees of Freedom Methodology, the Way to Control Law

Although the physical system has been identified and the global model derived, implementation into the final control algorithm won't follow the regular MPC framework. As already mentioned in the introduction, main lead will be final control simplicity. In order to utilize a well-established PID control, which is sufficiently applicable in the narrow range around the predefined operating point, and in a same time by having known the global process pattern, we form robust control structure presented by the block diagram of the Fig. 8. In the previous section derived the *Fuzzy Explicit Model* (FEM) will be enriched by integration block and expressed in the common transfer function

$$G_{FEM}(z) = \boldsymbol{F}(\hat{\boldsymbol{x}}(k))\frac{T_S}{T_a(z-1)}.$$

Inputs assigned by $N_1(z), N_2(z)$ denote the noise signal supplemented to the manipulated variable and measured output respectively. The $\boldsymbol{\chi} = [v_o,\ i_L,\ i_R,\ E]$ is a vector of measured process variables integrated into the control concept and providing the fuzzy model's tracking lead.

By forming the all SISO possible closed loop transfer functions from the control structure on the Fig. 8, assuming that other inputs are 0 and $d\boldsymbol{\chi}/dt = 0$, one can easily examine the existence of only two independent. Hence, it defines our control structure as the Two Degrees of Freedom control [24].

This control methodology complies with our main goal of partitioning standard MPC method and allowing independent adjustment of the system's response, linked up to the process constraints and steady state stability. Following the above methodology, controller consists of the steady-state fuzzy-model in the feed-forward line and the optimized PI controller in the main controller's line.

The control law is formed from the two, in a phase of designing, non-correlated control signals

$$d_u(t) = \frac{1}{T_a}\int d_{FEM}(kT_S)dt + d_{PI}(t) \tag{18}$$

where

$$d_{FEM}(k) = \mathbf{F}(\hat{\boldsymbol{x}}(k)) = \mathbf{F}\left(s(k),\, \hat{I}_L,\, E(k),\, \frac{\bar{v}_o(k)}{\bar{i}_R(k)}\right) \tag{19}$$

is the predictive part of the control algorithm based on the *Explicit Fuzzy Model* derived from the offline identification process in Section 3, and $d_{PI} = G_{PI}(s - v_o)$ is the output from the analogue PI controller.

Conceptually similar to the *Explicit Model Predictive Controls* [21] in the sense of the offline identified model based control, but advanced in minimization of the online computation complexity, this method opens up ability to conciliate a better controller's performance with avoidance of the complex *Mutiparametric Programming*. This method doesn't solve the standard predictive control problem and it is not based

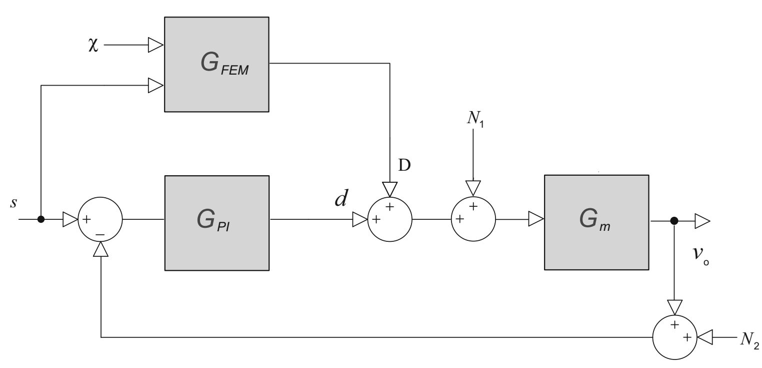


Fig. 8 Implementation of the Fuzzy Explicit Model in the typical two degrees of freedom control structure

on receding horizon principle, thus in correlation with standardized MPC methods points out:

- *The prediction horizon is infinite, it goes for an open-loop stable system and the prediction is related to the system's steady state.*
- *The reference trajectory is implemented by an extra integration on the output of the explicit model. The time of integration is a tuning parameter and affects the controller's aggressiveness in the transient time.*
- *The internal model is the Fuzzy Explicit Model of the steady-state duty cycle and not the Fuzzy Dynamical Model, and accordingly it does not suffer a typical feedback problem.*
- *The steady-state error is compensated with a standard analogue PI control tuned in the highest process gain regime and it is not treated by the predictive control itself.*
- *The feed-forward characteristic of the standard MPC method is explicitly fulfilled by the controller's configuration.*

For the simulation purpose of this work, shown in Fig. 9, and without loss of generality, the optimized PI controller will be constructed purely based on the MATLAB tools for the SISO controllers [22]. The original boost DC-DC converter or process in this investigation is shown in Fig. 1, will be linearized by the well-known perturbation method around the operating point, and accordingly as an "Averaged-Switch Model" [11] introduced in the control optimization of the PI controller.

The construction and tuning of the PI controller is done in the two standard steps of the "sisotool" MATLAB toolbox:

1. Construction of the PI controller by the auto-tuning method based on the singular frequency and minimizing the ITAE (Integral Time Absolute Error) performance
2. Optimization-based tuning by the Gradient Descent Algorithm for a Medium Scale.

The transfer function of the analogue controller in its equivalent discrete form for a sample time $t_{sample} = 10^{-6}\ s$ is

$$G_{PI}(z) = 5.648 \cdot 10^{-4} \left(\frac{z - 1.000354}{z - 1} \right). \tag{20}$$

The offline optimization is done around the operating point

$$s = 50\ V, \quad E = 10\ V, \quad R = 12.5\ \Omega \tag{21}$$

which is selected closer to the top border of the process gain and the coil current range in order to achieve a stable operation even with a large difference in the process parameters.

Optimization is performed on the "averaged-switch model" linearized around the operating point and in its discrete form

$$G_m(z) = \frac{-0.090237z + 0.0904}{z^2 - 2z + 0.9996}. \tag{22}$$

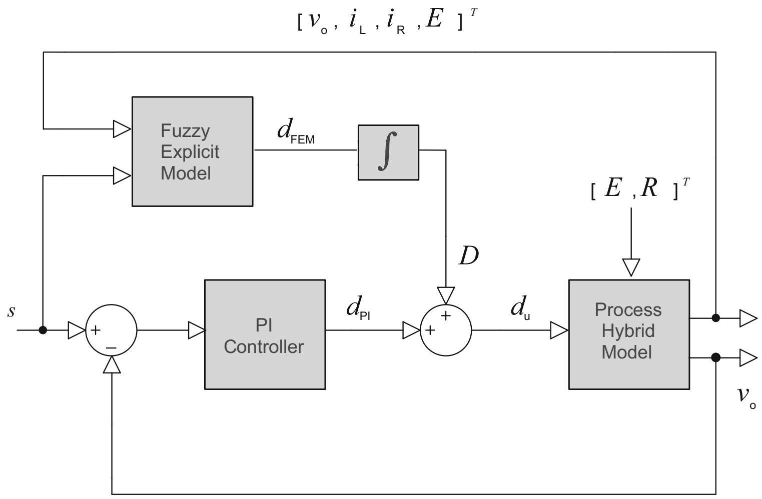


Fig. 9 Simulation model for the TDOF Control method based on Global and Explicit Fuzzy Model of the converter's steady-state duty cycle

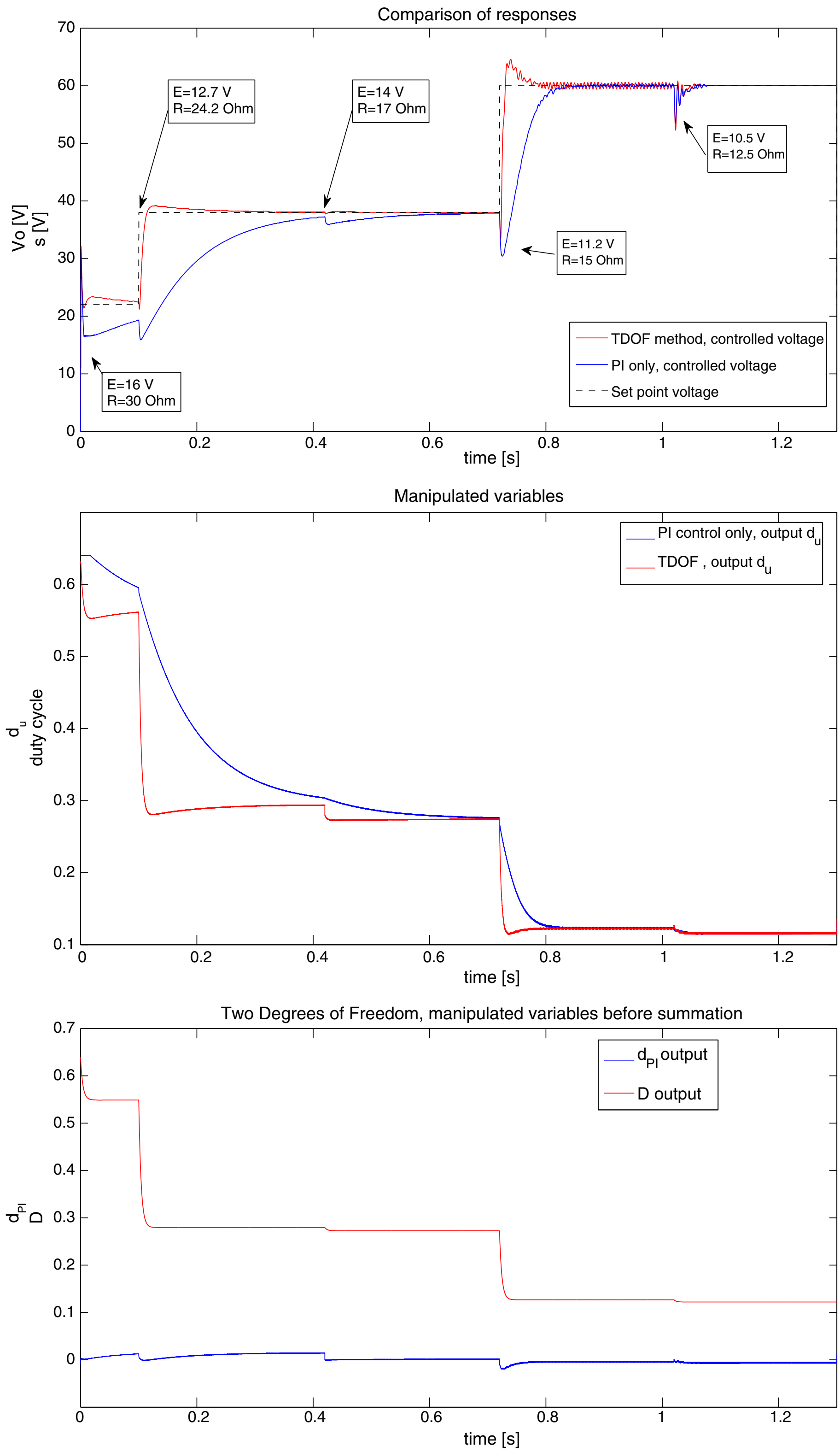


Fig. 10 Simulation results: a - Controlled variable v_o compared in between optimized PI and TDOF controller including FEM b - Manipulated variable d_u compared in between optimized PI and TDOF controller c - Manipulated variables of TDOF controller before the summation point in the control loop

From equation (19) it is transparent that the input in the fuzzy model is a vector of the measured values, except the one related to the predicted coil current in the steady state $\hat{I}_L$.

The steady-state coil current is calculated by involving the conserved energy law and the assumption that the load in the secondary circuit of the DC-DC converter will be changed only in a time that is incomparably wider than the scan time or $\Delta t >> T_S$. Also, the current efficiency factor has been taken as an average for a particular converter's operating range, already predefined with universes of discourses.

4.1 Simulation of the Control Algorithm

The simulation of the control algorithm involves a continuous disturbance, which in its final meaning has to result in the performance comparison of the TDOF method and the classic optimized PI control. The objectives in this work are primarily the robustness and minimization of the transient time. So, the process step parameter changes are commenced in combination, or all together for a wider operating range of the DC-DC converter. By altering the reference point s, the converter will be guided from the current discontinuous mode of operation to the continuous mode, where the highest process gain is expected at the top border of the coil current universe of discourse. The process parameters in the simulation are

$$T_S = 333.33\,\mu s,\quad L = 208\,\mu H,\quad C = 222\,\mu F.$$

The aggressiveness of the model control is tuned and resulted in

$$T_a = 0.004$$

by taking care of the current i_L constraint and the duty cycle d_u first derivation constraint.

Figure 10a shows the controlled process output responses for a certain controller on the step changes of the source voltage E, the load resistance R and the voltage set-point s. The optimized PI controller is tested in the two control structures. First, as pure PI control with the feed-forward line disconnected. Second, as a PI control integrated into the complete TDOF control structure. In the same test, Fig. 10b shows the controllers' manipulated variables. Furthermore, Fig. 10c shows the manipulated variable of TDOF controller presented by its two constructive parts. That is explicitly presenting the main features of the TDOF control methodology.

The disturbance of the process parameters is synchronized with the set point change or separately to simulate a possibly realistic DC-DC converter's operating regime.

Generally, in this work the developed TDOF controller features stable and robust operation. We see that the two dynamics approach, also affirmed by the decomposition of the general controller parameters, fulfilled expectations and presents the remarkable results relative to the complexity of the design and the online processing time. The stability of the control method relies on the stability of the PI controller and that is not a chain related to the delays in the transient time, which is now only related to the physical constraints. The only drawback is a naturally present model/process error and its effect on the steady-state error, manifesting as an overshoot, but now less harmful than the perspective cause of the nonlinearities in the standard MPC methods.

The offline optimized PI controller is comparable in the process higher gain range where the optimization was done. The constraints handling of the input signal in the PI optimized controller can be achieved as well as with MPC controllers by the selection of the highest gain operating point. In the TDOF controller, this feature is already integrated into the steady-state fuzzy model; therefore, a proper tuning of the PI parameters preserves it.

5 Conclusions

We present an efficient, new, MPC method based on the TDOF principle for an open-loop stable hybrid system that is state measurable. Instead of focusing on the transient process characteristics, the method is pointing out the global process knowledge of the steady state. As shown, this knowledge is integrated into the explicit fuzzy model gained by the identification process. Each processor's scan time, controller predicts the steady-state duty cycle and by concerning the physical constraints adopts with the fastest transient time to the process parameters' change. The misfortune in the model/process approximation error is compensated by a small signal PI optimized controller, developed with the standard toolbox. The stability of the control system is related only to the stability of the feedback related and standard PI controller by taking into consideration that process parameters' change period is incomparable longer than the controller's scanning time.

Further examinations will be conducted in the direction of an adaptive steady-state current prediction, based on a measurement and followed by the complete nonlinear phenomena exclusion.

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